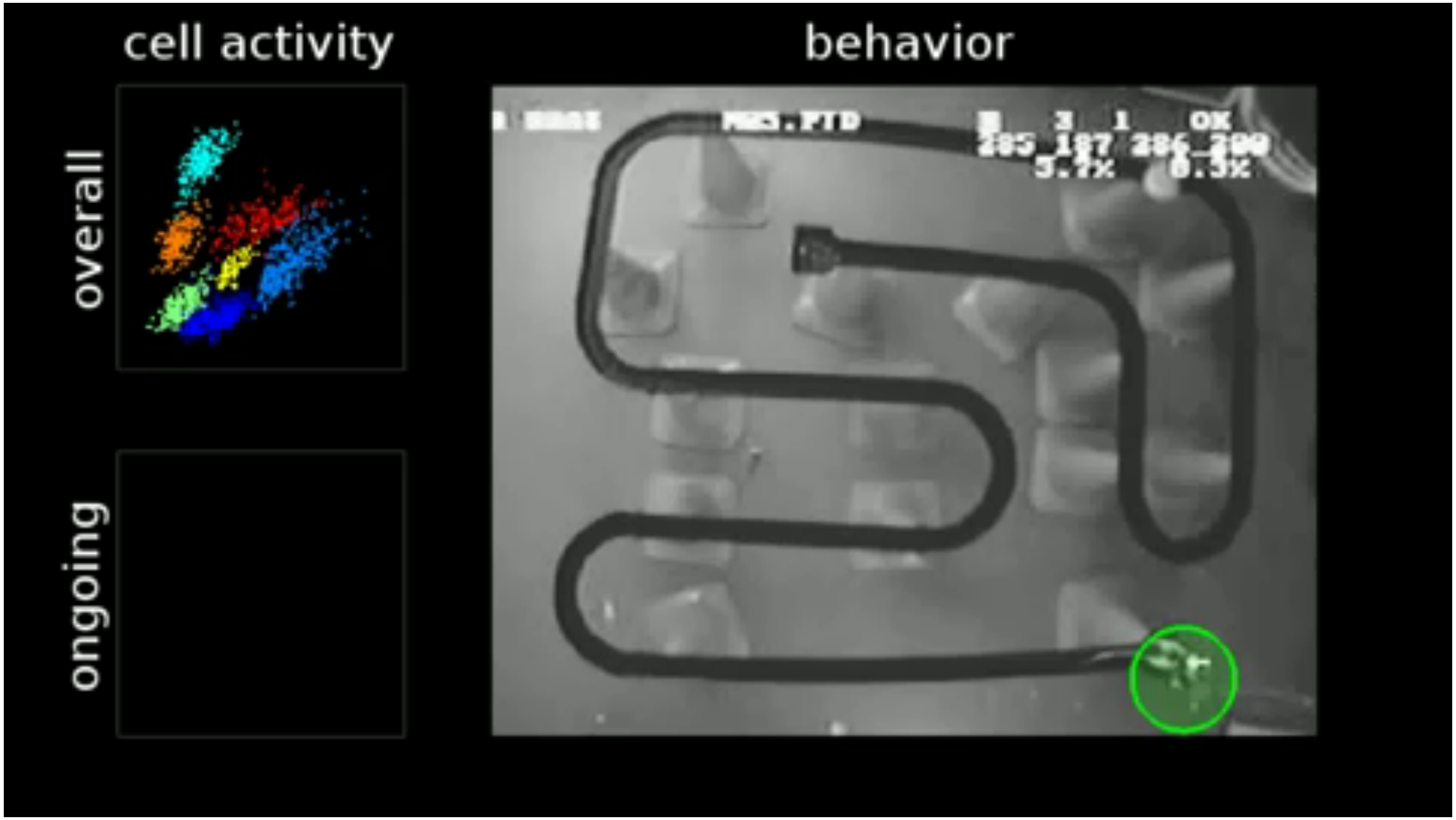
Modeling time-dependent system

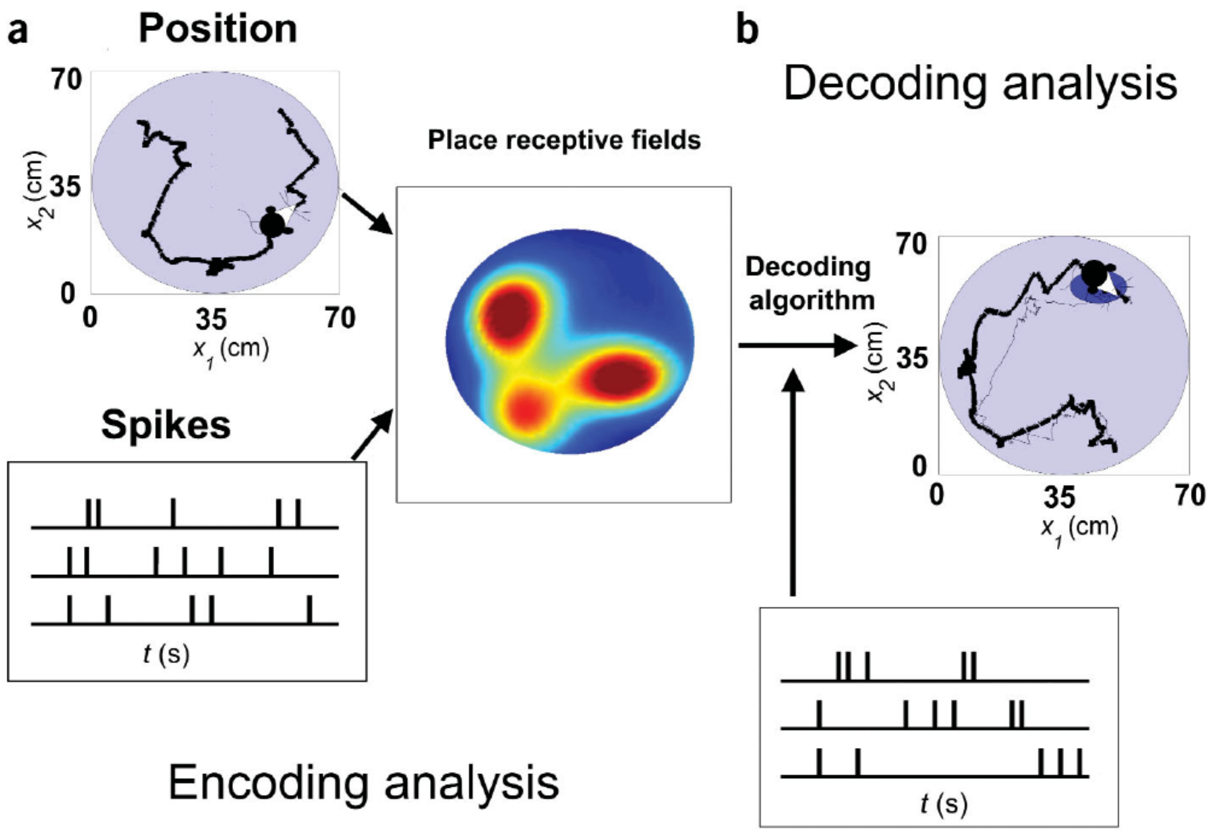
STATE SPACE MODEL

Demo



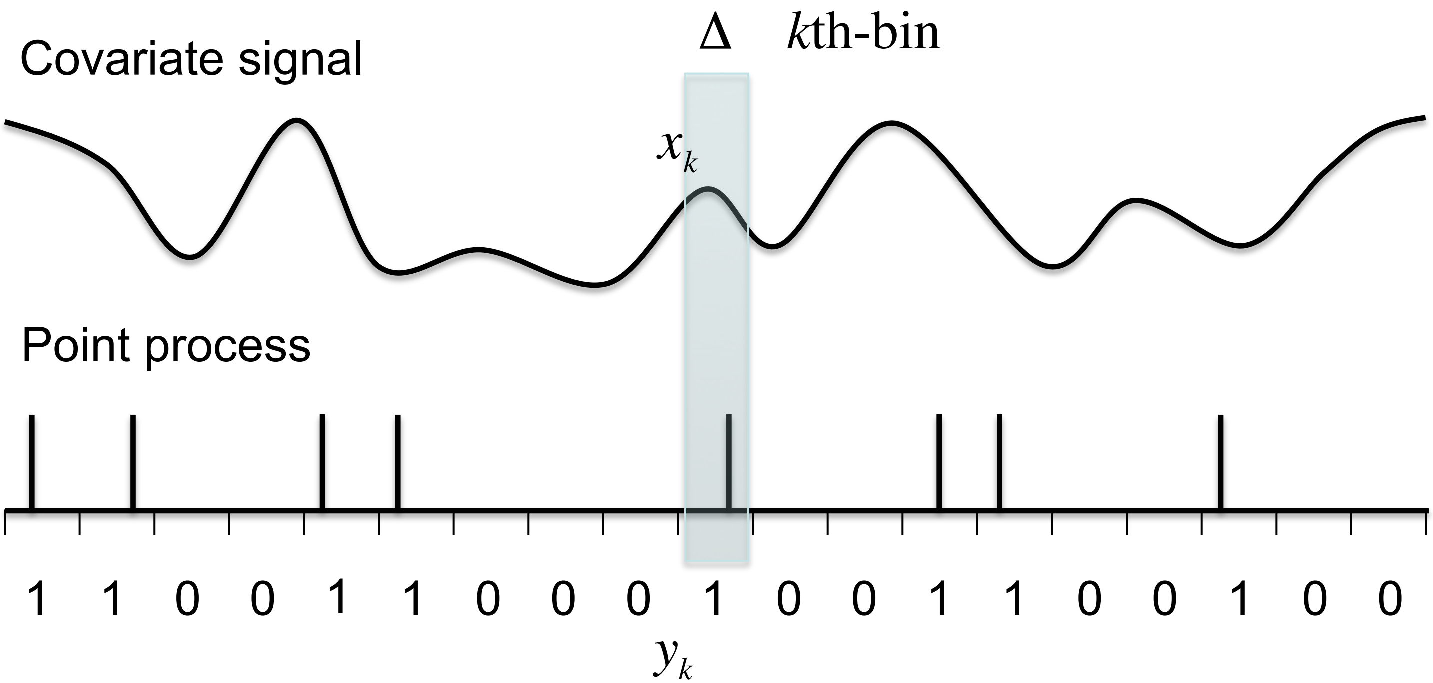
Hippocampal place cells recorded in the Wilson lab

Two-stages analysis of neural signal



Brown et al. Journal of Neuroscience 1998; 18: 7411-7425.

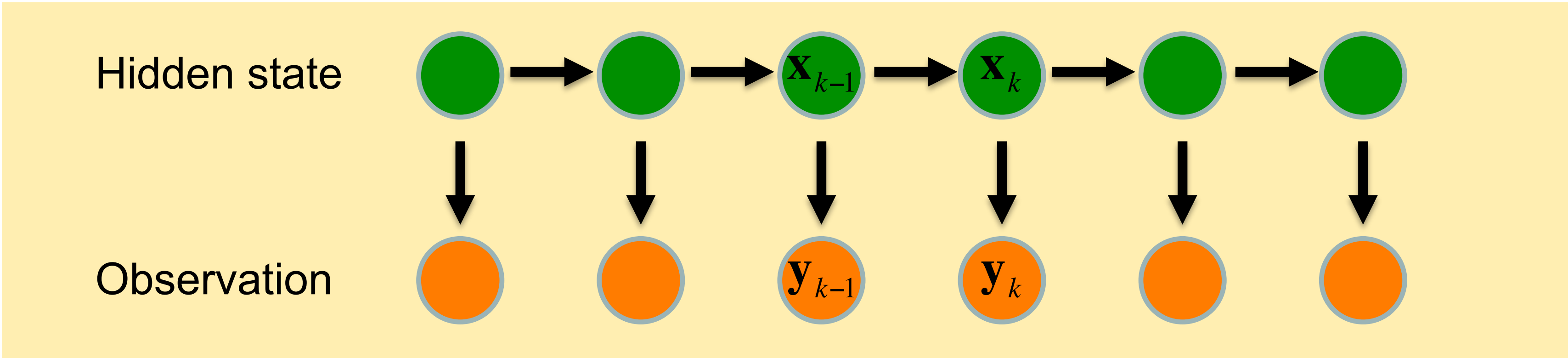
Discrete-time Poisson-GLM



$$P(y_k \mid \mathbf{x}_k, \beta) = \exp\left(y_k \log\left\{\lambda_k(\mathbf{x}_k, \beta)\Delta\right\} - \lambda_k(\mathbf{x}_k, \beta)\Delta\right)$$

$$\lambda_k(\mathbf{x}_k, \beta)\Delta = e^{\beta_0 + \mathbf{x}_k^T \beta}$$

State-space model



State model: Gaussian

$$\mathbf{x}_1 \sim N(\mu, \Sigma)$$

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \xi_k \qquad \xi_k \sim N(\mathbf{0}, \mathbf{Q})$$

Hyper-parameters:

$$\mathbf{w} = [\mathbf{Q}, \mu, \Sigma]$$

Observation model: Point process (discretized)

$$P(y_k \mid \mathbf{x}_k, \beta) = \exp\left(y_k \log\{\lambda_k(\mathbf{x}_k, \beta)\Delta\} - \lambda_k(\mathbf{x}_k, \beta)\Delta \right)$$

$$\lambda_k(\mathbf{x}_k, \beta)\Delta = e^{\beta_0 + \mathbf{x}_k^T \beta}$$

Posterior density of the state

Bayes' rule:

Posterior **Likelihood** **Prior (State model)**

$$p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right) = \frac{P\left(y_{1:K} \mid \mathbf{x}_{1:K}, \beta\right) p\left(\mathbf{x}_{1:K} \mid \mathbf{w}\right)}{P\left(y_{1:K} \mid \beta, \mathbf{w}\right)}$$

Evidence

$\beta, \mathbf{w}$ **parameters**

Once we construct the posterior, we can obtain

MAP estimate: the most likely path of the hidden state.

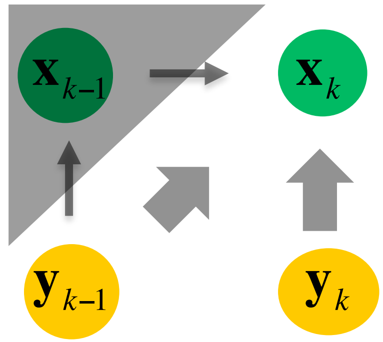
Credible interval: an analog of confidence interval in Bayesian estimation.

We obtain the joint posterior density => Bayesian recursive filter.

Recursive Bayesian filter

One-step prediction

$p\left(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}\right)$



Filter

$p\left(\mathbf{x}_{1:k} \mid \mathbf{y}_{1:k}\right)$

Filter at k-th step | **Likelihood** | **One-step prediction**

Bayes' rule

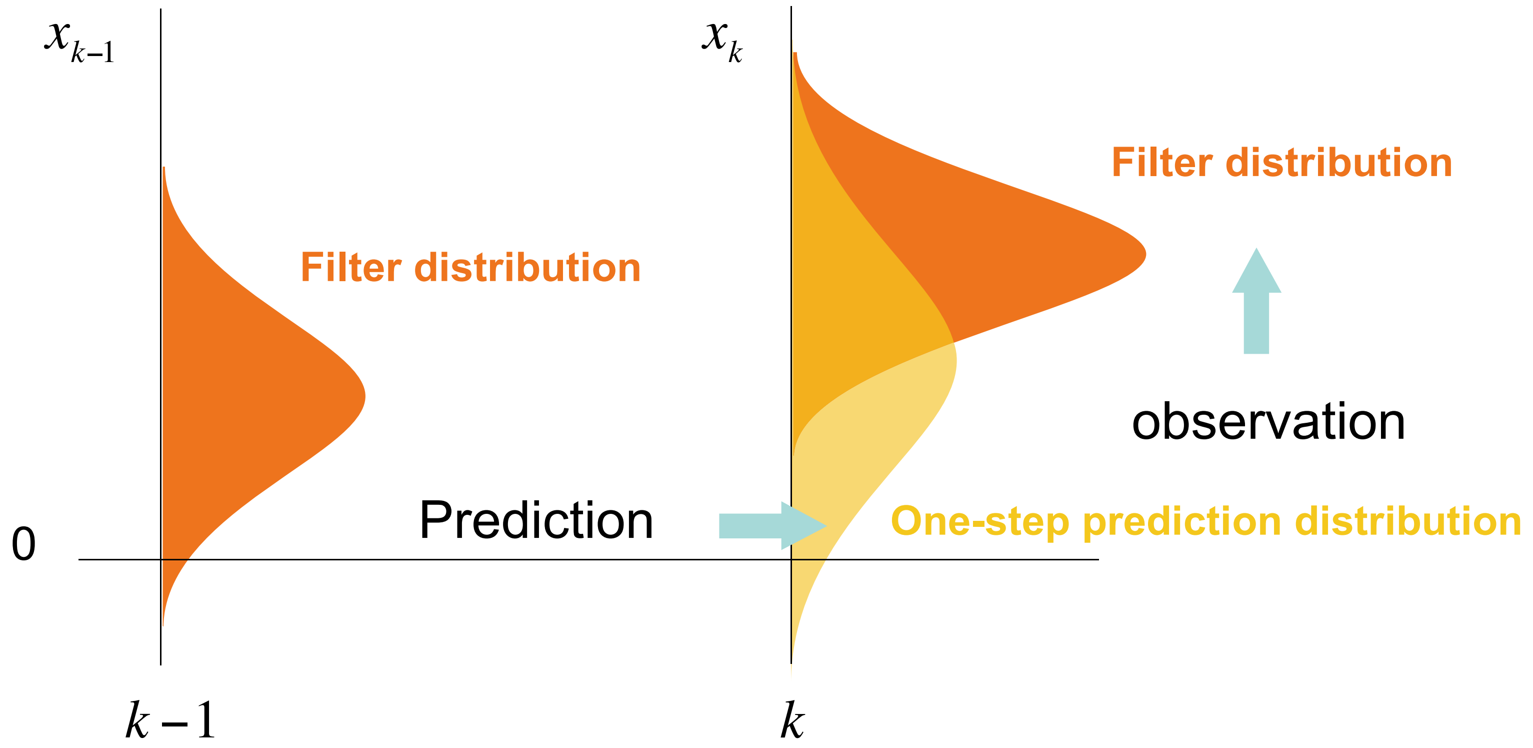
$$p\left(\mathbf{x}_k \mid y_{1:k}, \beta, \mathbf{w}\right) = \frac{P\left(y_k \mid \mathbf{x}_k, \beta\right) p\left(\mathbf{x}_k \mid y_{1:k-1}, \beta, \mathbf{w}\right)}{P\left(y_k \mid \mathbf{x}_{k-1}, \beta, \mathbf{w}\right)}$$

Chapman-Kolmogorov equation

$$p\left(\mathbf{x}_k \mid y_{1:k-1}, \beta, \mathbf{w}\right) = \int p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \mathbf{w}\right) \cdot p\left(\mathbf{x}_{k-1} \mid y_{1:k-1}, \beta, \mathbf{w}\right) d\mathbf{x}_{k-1}$$

Filter density at (k-1)-th step

Recursive Bayesian filter



Methods to obtain the posterior

Methods for obtaining the posterior density to name a few…

Analytical methods

Gaussian approximation (Laplace's method).

Conjugate prior.

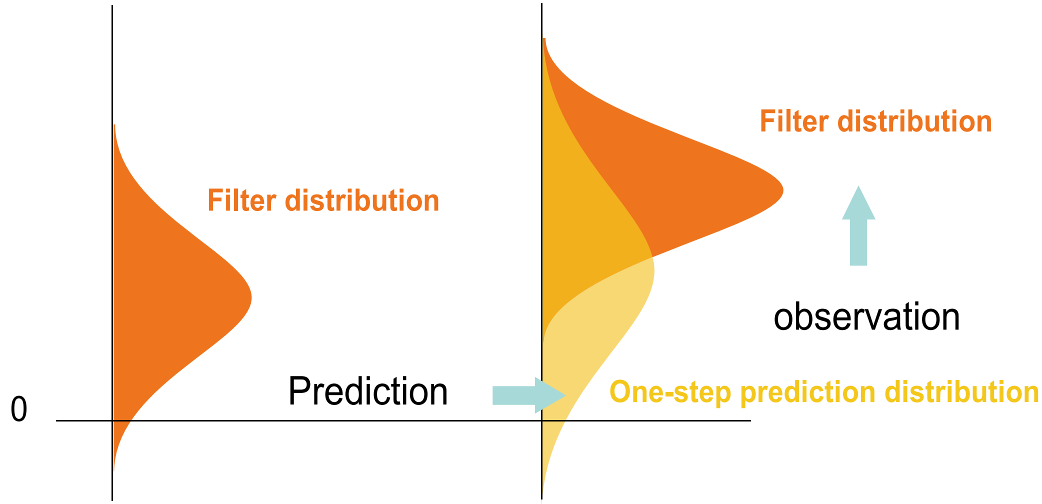
Expectation propagation.

Monte Carlo methods

Sequential importance resampling (Particle filter).

Markov chain Monte Carlo (MCMC).

One-step prediction density



Chapman-Kolmogorov equation

$$p(\mathbf{x}_k \mid y_{1:k-1}, \mathbf{w}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \mathbf{w}) \cdot p(\mathbf{x}_{k-1} \mid y_{1:k-1}, \beta, \mathbf{w})\, d\mathbf{x}_{k-1}$$

$$N(\mathbf{x}_{k-1}, Q) \qquad N(\mathbf{x}_{k-1|k-1}, W_{k-1|k-1})$$

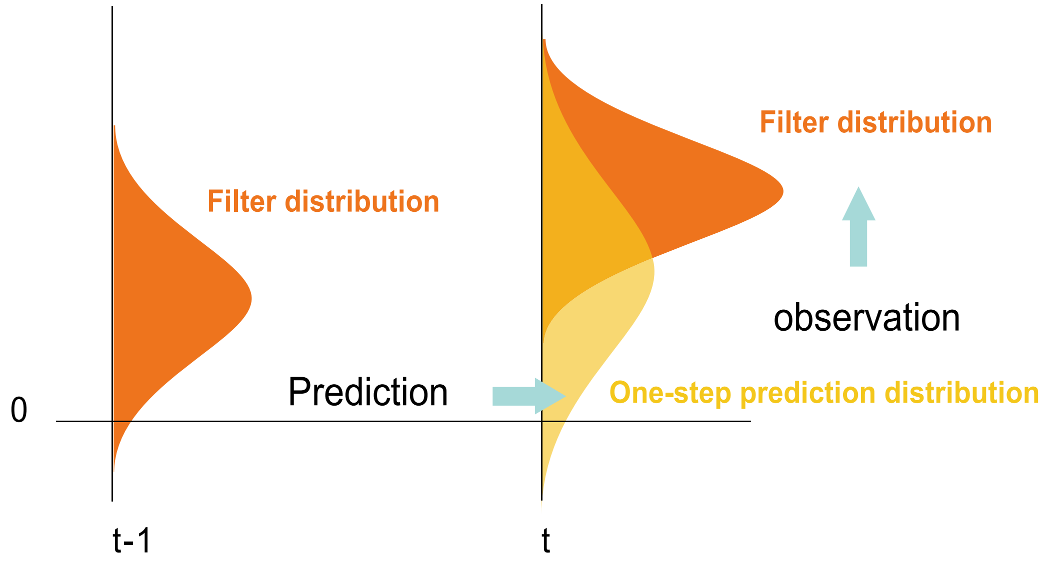
Assumption

One-step prediction is a normal density with

mean $\mathbf{x}_{k|k-1} = \mathbf{x}_{k-1|k-1}$

covariance $W_{k|k-1} = W_{k-1|k-1} + Q$

Filter density



Filter density **Likelihood** **One-step prediction density**

$$p\left(\mathbf{x}_k \mid y_{1:k}, \beta, \mathbf{w}\right) = \frac{P\left(y_k \mid \mathbf{x}_k, \beta\right) p\left(\mathbf{x}_k \mid y_{1:k-1}, \beta, \mathbf{w}\right)}{p\left(y_k \mid \mathbf{x}_{k-1}, \beta, \mathbf{w}\right)}$$

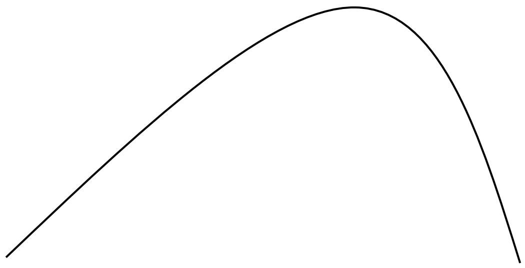
$$\propto \exp[y_k \log\left\{\lambda_k\left(\mathbf{x}_k, \beta\right)\Delta\right\} - \lambda_k\left(\mathbf{x}_k, \beta\right)\Delta$$

$$-\frac{1}{2}\left(\mathbf{x}_k - \mathbf{x}_{k|k-1}\right)^T W_{k|k-1}^{-1}\left(\mathbf{x}_k - \mathbf{x}_{k|k-1}\right)]$$

Log-concave functions

Rationale

Log-concave:
the function is log-concave if the logarithm of the function is concave.



Concave function.

Exponential family of distributions are in general log-concave
with respect to parameters in 'natural form'.

A function obtained by multiplication of two log-concave functions
is log-concave.

Log-concave ⬅ Log-concave Log-concave

$$p\left(\mathbf{x}_k \mid y_{1:k}, \beta, \mathbf{w}\right) \propto P\left(y_k \mid \mathbf{x}_k, \beta\right) p\left(\mathbf{x}_k \mid y_{1:k-1}, \beta, \mathbf{w}\right)$$

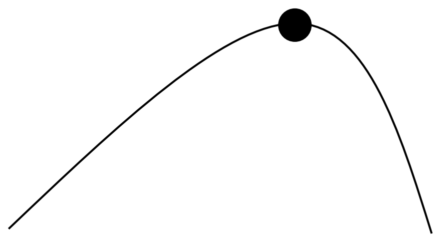
Paninski, L. (2005). Advances in NIPS 17 1025–1032
Boyd S. (2004). Convex Optimization

Newton-Raphson method

Newton-Raphson method to find a posterior mode

Posterior mode (MAP estimate):

$$\mathbf{x}_{k|k} \equiv \underset{\mathbf{x}_k}{\operatorname{argmax}} f\left(\mathbf{x}_k\right)$$



Log-posterior:

$$f\left(\mathbf{x}_k\right) = y_k \log\left(\lambda_k \Delta\right) - \lambda_k \Delta - \frac{1}{2}\left(\mathbf{x}_k - \mathbf{x}_{k|k-1}\right)^T W_{k|k-1}^{-1}\left(\mathbf{x}_k - \mathbf{x}_{k|k-1}\right)$$

Newton-Raphson method:

$$\mathbf{x}_k^{\text{new}} = \mathbf{x}_k^{\text{old}} - \left(\nabla\nabla f\right)^{-1} \nabla f$$

Hessian Gradient evaluated at $\mathbf{x}_k^{\text{old}}$

Gradient and Hessian

gradient $\nabla f\left(\mathbf{x}_k\right)=\left(y_k-\lambda_k\Delta\right)\frac{\partial\log\lambda_k}{\partial\mathbf{x}_k}-W_{k|k-1}^{-1}\left(\mathbf{x}_k-\mathbf{x}_{k|k-1}\right)$

Hessian $H\equiv\frac{\partial f\left(\mathbf{x}_k\right)}{\partial\mathbf{x}_k\partial\mathbf{x}_k^T}=-\frac{\partial\log\lambda_k}{\partial\mathbf{x}^T}\lambda_k\Delta\frac{\partial\log\lambda_k}{\partial\mathbf{x}_k}+\left(y_k-\lambda_k\Delta\right)\frac{\partial\log\lambda_k}{\partial\mathbf{x}_k\partial\mathbf{x}_k^T}-W_{k|k-1}^{-1}$

If we use a canonical link function $\lambda_k=e^{\beta_0+\beta\mathbf{x}_k}$

gradient $\nabla f\left(\mathbf{x}_k\right)=\left(y_k-\lambda_k\Delta\right)\beta-W_{k|k-1}^{-1}\left(\mathbf{x}_k-\mathbf{x}_{k|k-1}\right)$

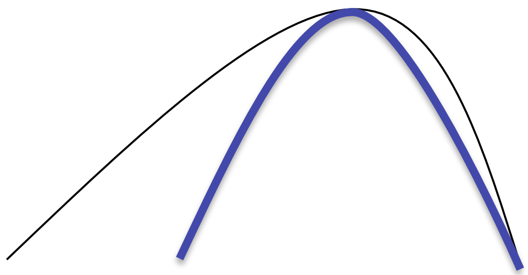
Hessian $H=-\beta^T\lambda_k\Delta\beta-W_{k|k-1}^{-1}$

Gaussian approximation

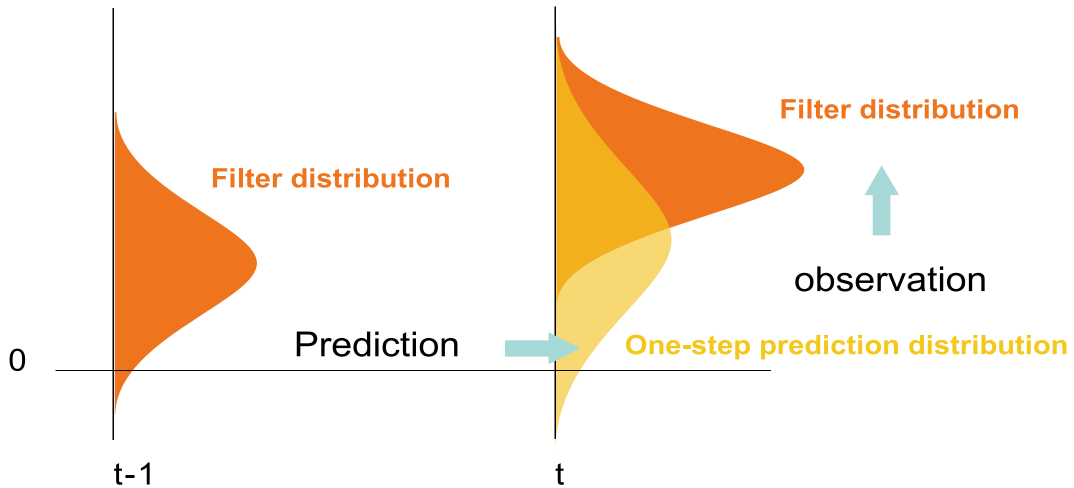
Laplace's method for a Gaussian approximation

Filter mean: $\mathbf{x}_{k|k}$ $\qquad \nabla f\left(\mathbf{x}_k\right) = 0$

Filter covariance: $W_{k|k}$ $\qquad W_{k|k} = -H^{-1}\Big|_{\mathbf{x}_{k|k}}$



Recursive nonlinear filtering



Recursive nonlinear filtering

One-step prediction

Filter mean: $$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + W_{k|k-1}\beta\left(y_k - \lambda_k\left(\mathbf{x}_{k|k}\right)\Delta\right)$$

Solved by Newton-Raphson Method

Observation error (innovation)

Filter covariance: $$W_{k|k}^{-1} = W_{k|k-1}^{-1} + \beta^T \lambda_k\left(\mathbf{x}_{k|k}\right)\Delta\beta$$

One-step prediction   Information given by data at k-th step.

Filtering/Smoothing

Forward recursion $k = 1, 2, \cdots, K$

One-step prediction

Mean $\mathbf{x}_{t|t-1} = \mathbf{x}_{t-1|t-1}$

Covariance $W_{t|t-1} = W_{t-1|t-1} + \mathbf{Q}$

Recursive nonlinear filtering

Mean $\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + W_{k|k-1}\beta\left(y_k - \lambda_k\left(\mathbf{x}_{k|k}\right)\Delta\right)$

Solved by
Newton-Raphson Method

Covariance $W_{k|k}^{-1} = W_{k|k-1}^{-1} + \beta^T \lambda_k\left(\mathbf{x}_{k|k}\right)\Delta\beta$

Backward recursion $k = K-1, K-2, \cdots, 2, 1$

Fixed interval smoothing

Mean $\mathbf{x}_{k|K} = \mathbf{x}_{k|k} + A_k\left[\mathbf{x}_{k+1|K} - \mathbf{x}_{k+1|k}\right]$ $A_k = W_{k|k} W_{k+1|k}^{-1}$

Covariance $W_{k|K} = W_{k|k} + A_k\left[W_{k+1|K} - W_{k+1|k}\right]A_k^T$

Joint estimation of posterior and parameters

Expectation-Maximization algorithm for joint estimation of posterior and parameters. (ref. Smith & Brown, 2003)

E-step: Given the parameters, construct the posterior

Posterior **Likelihood** **Prior**

$$p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right) = \frac{P\left(y_{1:K} \mid \mathbf{x}_{1:K}, \beta\right) p\left(\mathbf{x}_{1:K} \mid \mathbf{w}\right)}{P\left(y_{1:K} \mid \beta, \mathbf{w}\right)}$$

Evidence (marginal likelihood)

M-step: Given the posterior, optimize the parameters

Optimization of the parameters bymaximizing the (marginal) likelihood.

$$\mathbf{w}_{\mathrm{MLE}} = \operatorname*{argmax}_{\mathbf{w}} \log P\left(y_{1:K} \mid \beta, \mathbf{w}\right)$$

For this goal, we maximize a lower bound of the marginal likelihood (Q-function)

$$Q\left(\mathbf{w}^{\mathrm{new}} \mid \mathbf{w}\right) \equiv E_{\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}}\left[\log P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\mathrm{new}}\right)\right]$$

Expected complete data log-likelihood.

Derivation of the lower bound

The marginal log-likelihood function can be bounded as follows.

$$
\begin{aligned}
&\log P\left(y_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right) \\
&= \log \int P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right) d\mathbf{x}_{1:K} \\
&= \log \int p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right) \frac{P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right)}{p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right)} d\mathbf{x}_{1:K} \\
&= \log E_{\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}} \left[\frac{P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right)}{p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right)} \right] \\
&\geq E_{\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}} \left[\log \frac{P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right)}{p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right)} \right] \\
&= E_{\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}} \left[\log P\left(y_{1:K}, \mathbf{x}_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right) \right] - E_{\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}} \left[\log p\left(\mathbf{x}_{1:K} \mid y_{1:K}, \beta, \mathbf{w}\right) \right]
\end{aligned}
$$

Entropy of the posterior

Q-function

Irrelevant to $\mathbf{w}^{\text{new}}$

Hence we obtain

$$
\log P\left(y_{1:K} \mid \beta, \mathbf{w}^{\text{new}}\right) \geq Q\left(\mathbf{w}^{\text{new}} \mid \mathbf{w}\right)
$$

Model selection

$$p\left(\mathbf{y}_{1:T}\right) = \int p\left(y_{1:T}, \mathbf{x}_{1:T}\right) d\mathbf{x}_{1:T}$$

Data **Data** **Hidden state**

Bayesian model selection

Bayes Factor $$B_{12}\left(\mathbf{y}_{1:T}\right) = \frac{p\left(\mathbf{y}_{1:T} \mid M_1\right)}{p\left(\mathbf{y}_{1:T} \mid M_2\right)}$$

(Jeffrey 61, Kass&Raftery 95)

Model Selection penalized by model dimension

$$\text{ABIC} = -2\log \int p\left(y_{1:T}, \mathbf{x}_{1:T} \mid \mathbf{w}\right) d\mathbf{x}_{1:T} + 2 \times \text{model dimension}$$

Log-quadratic approximation

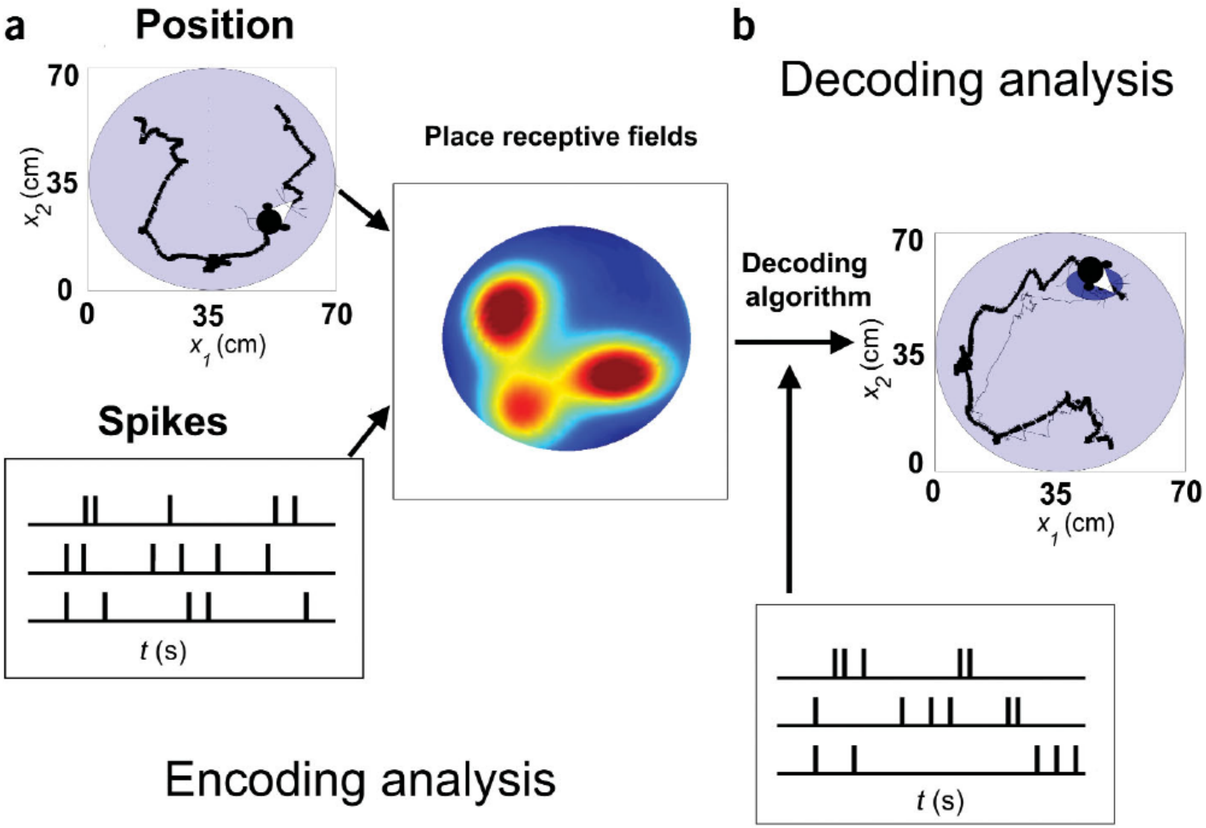
APPLICATIONS OF THE STATE-SPACE MODEL

The Journal of Neuroscience, September 15, 1998, 18(18):7411–7425

A Statistical Paradigm for Neural Spike Train Decoding Applied to Position Prediction from Ensemble Firing Patterns of Rat Hippocampal Place Cells

Emery N. Brown,[1] Loren M. Frank,[2] Dengda Tang,[1] Michael C. Quirk,[2] and Matthew A. Wilson[2]

Two stages analysis



Encoding model

Likelihood function

$$f(\mathbf{I}(t_k)|x(t_k), t_{k-1}) = \prod_{c=1}^{C} \{[\lambda^c[x(t_k)]\Lambda^c[\theta(\Delta_k)]]^{I_c(t_k)} \exp\{-\lambda^c[x(t_k)]\Lambda^c[\theta(\Delta_k)]\};$$

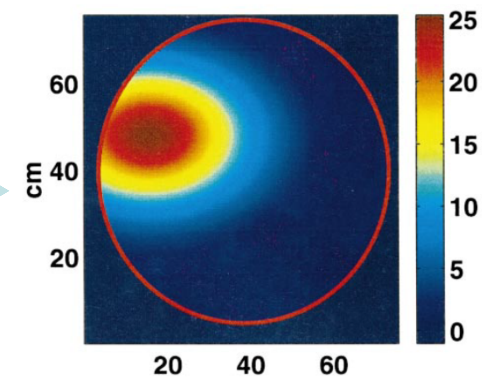
Conditional intensity function

$$\lambda^c(t|x(t), \phi(t), \xi_x^c) = \lambda_x^c(t|x(t), \xi_x^c)\lambda_\theta^c(t|\phi(t), \xi_\theta^c)$$

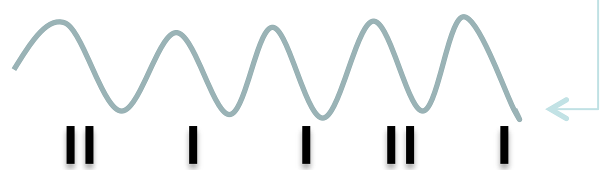
$$\lambda_x^c(t|x(t), \xi_x^c) = \exp\left\{ \alpha_c - \frac{1}{2}(x(t) - \mu_c)' W_c^{-1}(x(t) - \mu_c) \right\}$$

$$\lambda_\theta^c(t|\phi(t), \xi_\theta^c) = \exp\{\beta_c \cos(\phi(t) - \phi_c)\}$$

Hippocampal place cell receptive field model

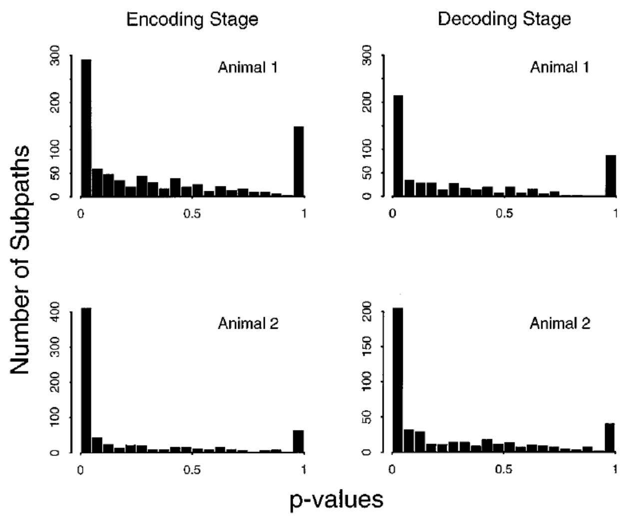


Theta phase modulation of spike rate



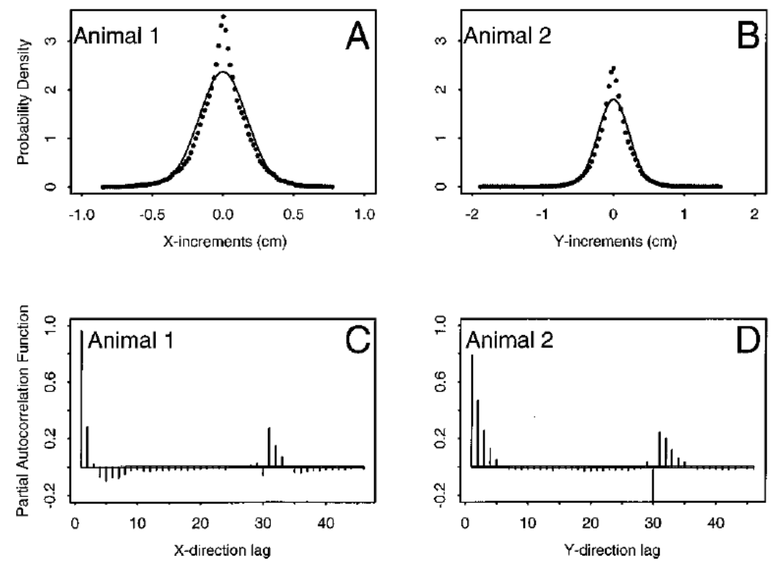
Goodness-of-fit of the encoding model

p-values of observed spike count against a null hypothesis of a Poisson model



Cells are more variable than a Poisson

Residual analysis of the state-model



Test rejected a bivariate Gaussian model.
Assumption of independence was also rejected.

Recursive point-process filter

Posterior (Filter)

Data up to time t_k

$$\Pr(x(t_k)|spikes\ in\ (t_e,\, t_k]) =$$

Observation

Data up to time t_k-1

$$\frac{\Pr(x(t_k)|spikes\ in\ (t_e,\, t_{k-1}]) \times \Pr(spikes\ at\ t_k|x(t_k),\, t_{k-1})}{\Pr(spikes\ at\ t_k|spikes\ in\ (t_e,\, t_{k-1}])}$$

One-step prediction

$$\Pr(x(t_k)|spikes\ in\ (t_e,\, t_{k-1}])$$

$$= \int \Pr(x(t_{k-1})|spikes\ in\ (t_e,\, t_{k-1}]) \times \Pr(x(t_k)|x(t_{k-1}))dx(t_{k-1})$$

Filter at t_k-1

State equation $x(t_k) - x(t_{k-1}) \sim N(0,\, W_x(\Delta_k))$

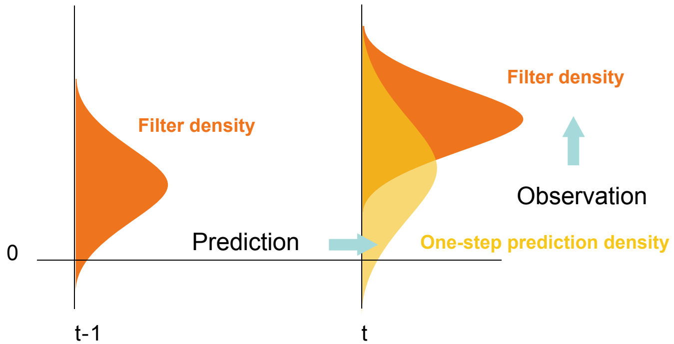
Algorithm for a point process filter

One-step prediction equation:

$$\hat{x}(t_k|t_{k-1}) = \hat{x}(t_{k-1}|t_{k-1});$$

One-step prediction variance:

$$W(t_k|t_{k-1}) = W_x(\Delta_k) + W(t_{k-1}|t_{k-1});$$



Posterior mode

$$\hat{x}(t_k|t_k) = [W(t_k|t_{k-1})^{-1} + \sum_{c=1}^{C} A_c[\hat{x}(t_k|t_k), \theta(\Delta_k)]W_c^{-1}]^{-1}$$

$$\times [W(t_k|t_{k-1})^{-1}\hat{x}(t_k|t_{k-1}) + \sum_{c=1}^{C} A_c[\hat{x}(t_k|t_k), \theta(\Delta_k)]W_c^{-1}\mu_c];$$

$$A_c[x(t_k|t_k), \theta(\Delta_k)] = I_c(t_k) - \lambda^c[x(t_k|t_k)]\Lambda^c[\theta(\Delta_k)];$$

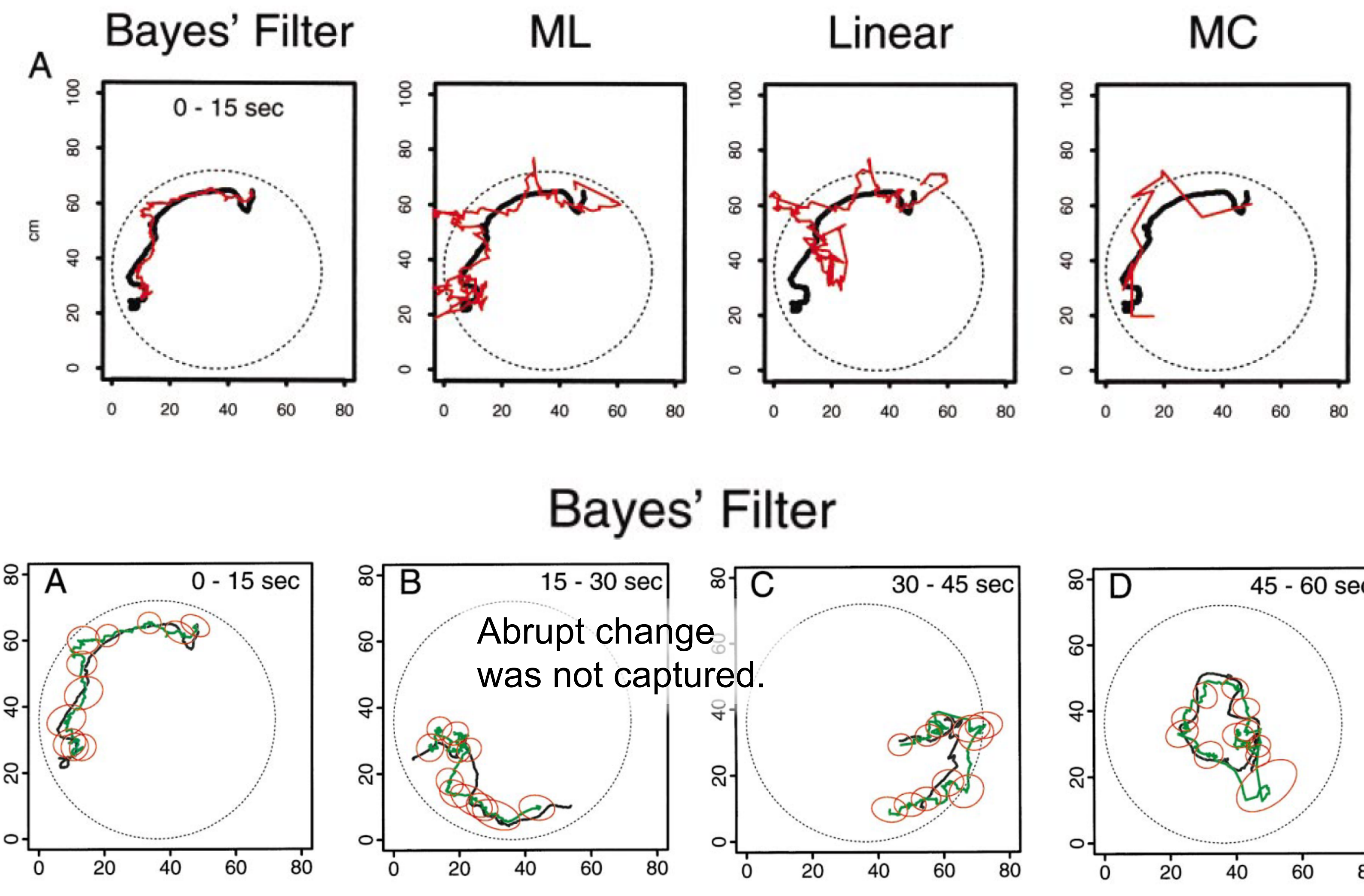
$$\Lambda^c[\theta(\Delta_k)] = \int^{t_k} \exp\{\beta_c \cos(\phi(t) - \phi_c)\}dt,$$

Posterior variance:

$$W(t_k|t_k) = [W(t_k|t_{k-1})^{-1} + \sum_{c=1}^{C} A_c[\hat{x}(t_k|t_k), \theta(\Delta_k)]W_c^{-1}$$

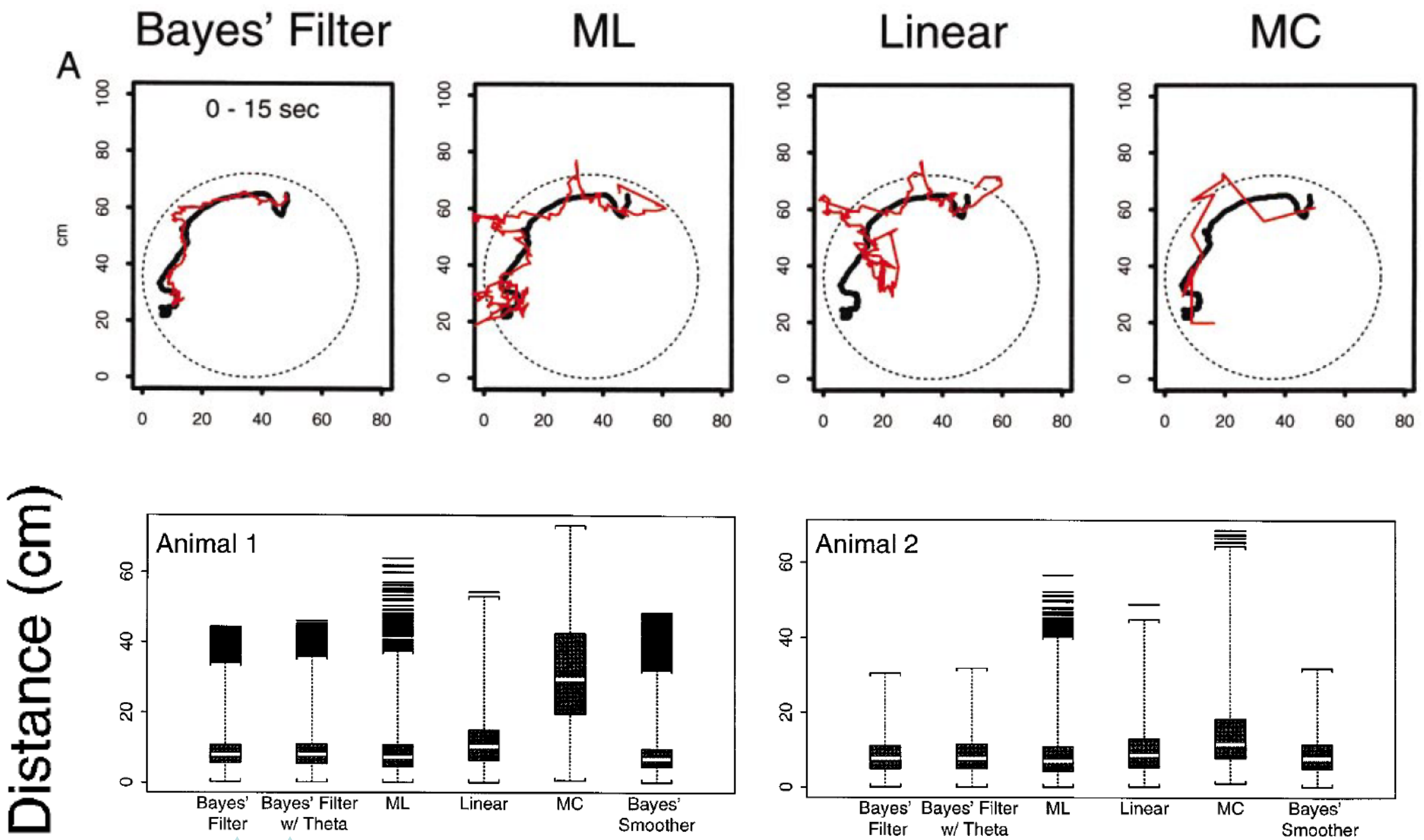
$$+ \sum_{c=1}^{C} \lambda^c[\hat{x}(t_k|t_k)]\Lambda^c[\theta(\Delta_k)]W_c^{-1}(\hat{x}(t_k|t_k) - \mu_c)(\hat{x}(t_k|t_k) - \mu_c)' W_c^{-1}]^{-1}$$

DECODING



TIME

DECODING



No difference.

ML was the second best, however with a lot of outliers.

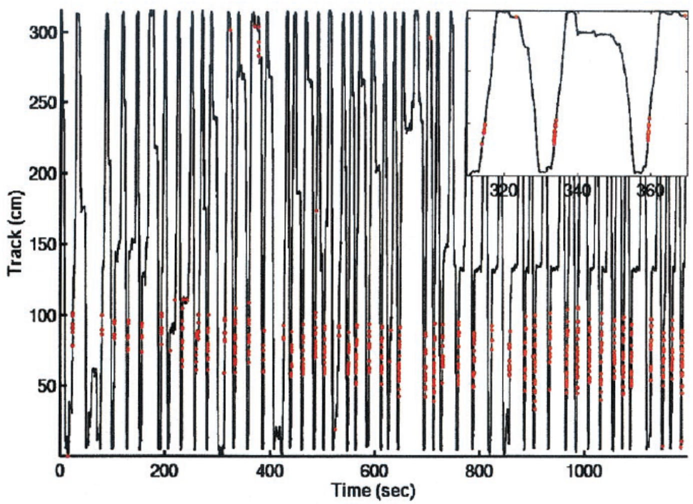
An analysis of neural receptive field plasticity by point process adaptive filtering

PNAS | October 9, 2001 | vol. 98 | no. 21 | 12261–12266

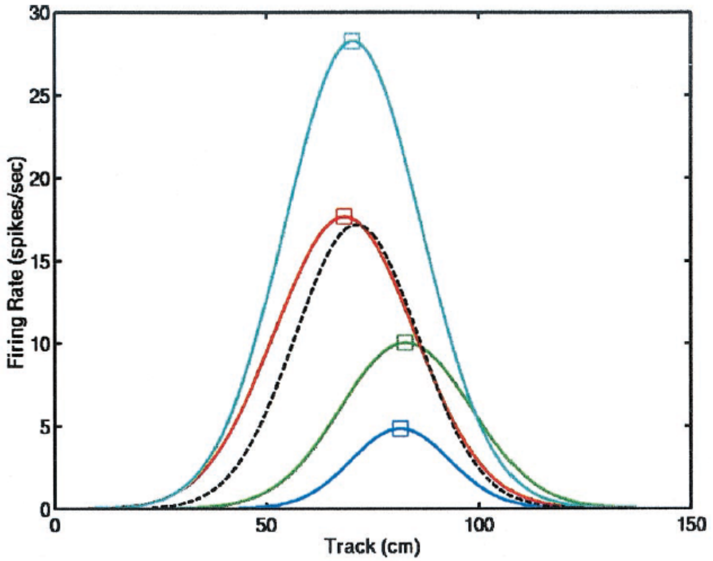
Emery N. Brown*†‡, David P. Nguyen*, Loren M. Frank*†, Matthew A. Wilson§, and Victor Solo¶

1D animal position

spikes



Change in the receptive fieldå



Conditional intensity model

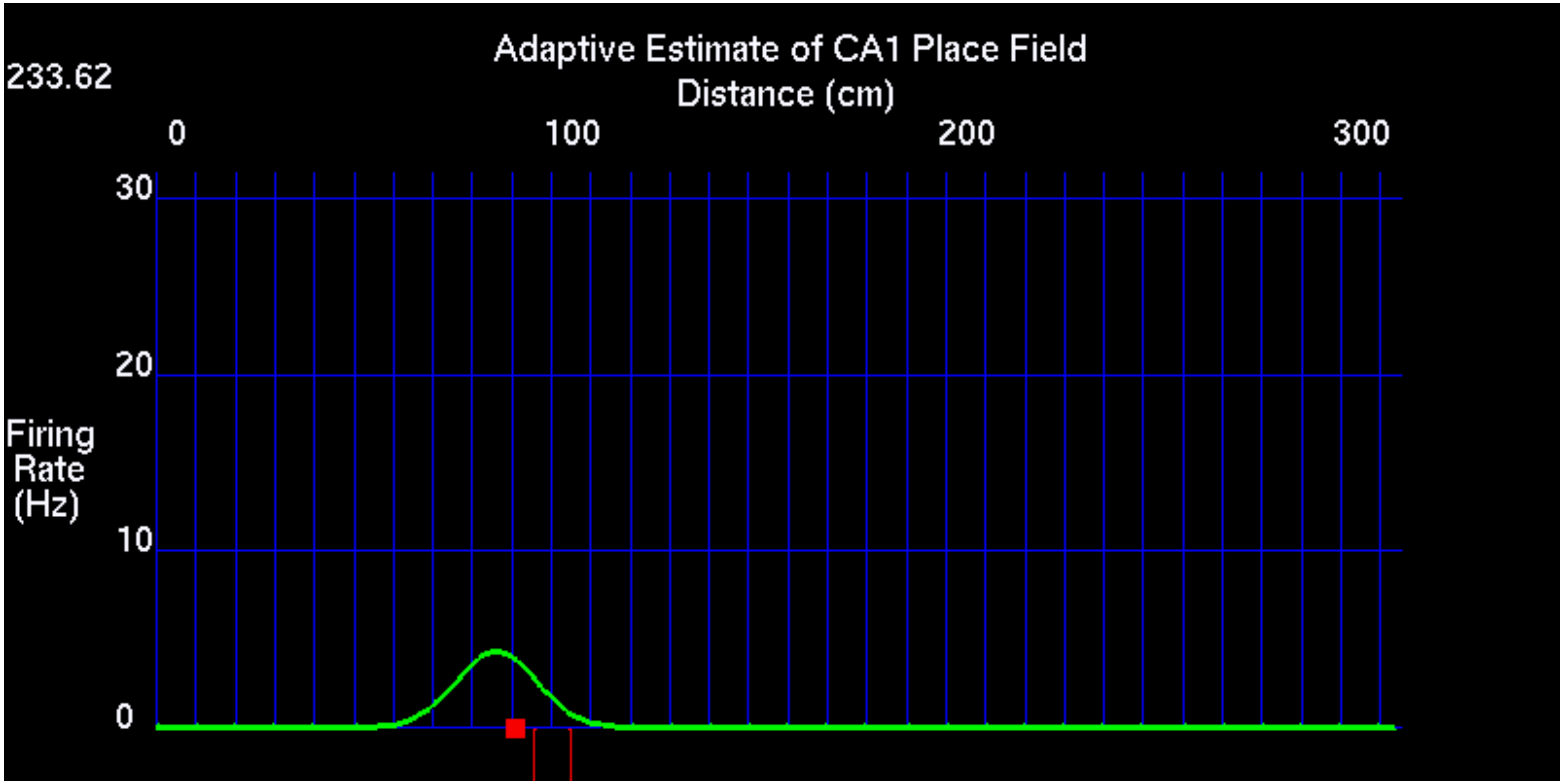
$$\lambda(t|\theta) = \exp\left\{\alpha - \frac{(x(t) - \mu)^2}{2\sigma^2}\right\}$$

$$\theta = (\alpha,\ \sigma,\ \mu)'$$

Update rule

$$\hat{\theta}_k = \hat{\theta}_{k-1} - \varepsilon \frac{\partial l_k(\theta)}{\partial \theta}\bigg|_{\theta = \hat{\theta}_{k-1}}$$

Demo



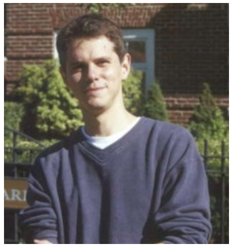
An analysis of neural receptive field plasticity by point process adaptive filtering

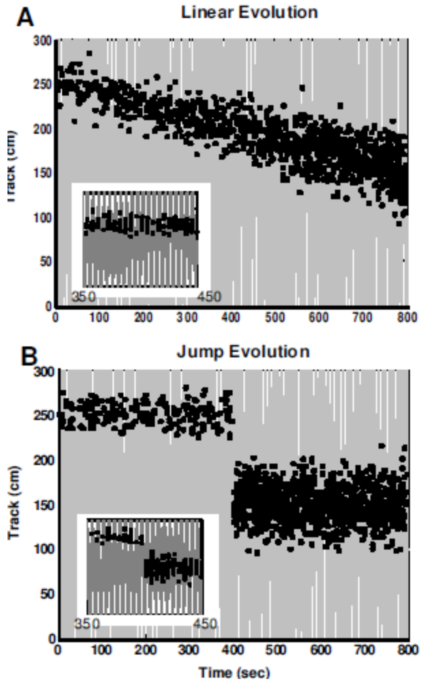
Emery N. Brown*†‡, David P. Nguyen*, Loren M. Frank*†, Matthew A. Wilson§, and Victor Solo¶

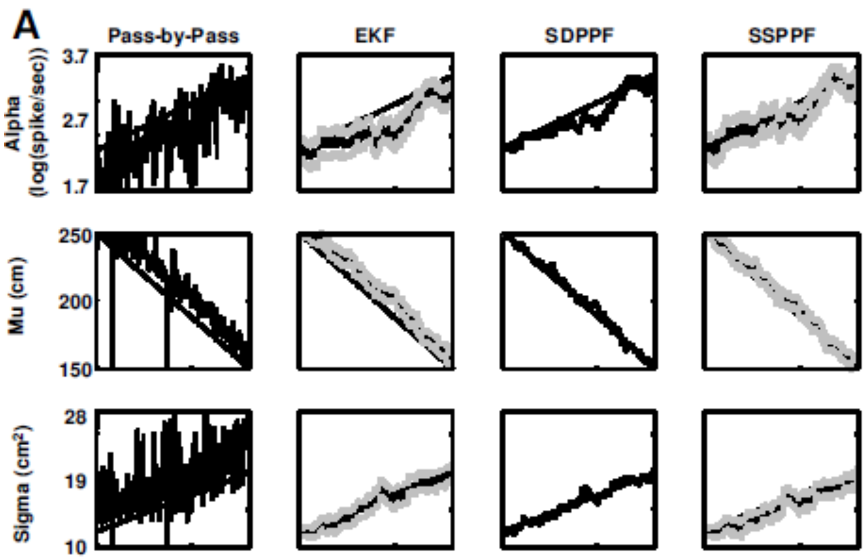
Neural Computation 16, 971–998 (2004)

Dynamic Analysis of Neural Encoding by Point Process Adaptive Filtering

Uri T. Eden Loren M. Frank Riccardo Barbieri Victor Solo Emery N. Brown







- A full mathematical formulation of an adaptive point process filter.
- Fast approximation of the adaptive point process filter.
- Simulation study on (1) tracking place filed dynamics, (2) simultaneous estimation of receptive field dynamics and arm trajectory (decoding).

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What we learned

1
- **Framework of a state-space model.**

2
- **Recursive Bayesian filter (Laplace's approximation).**

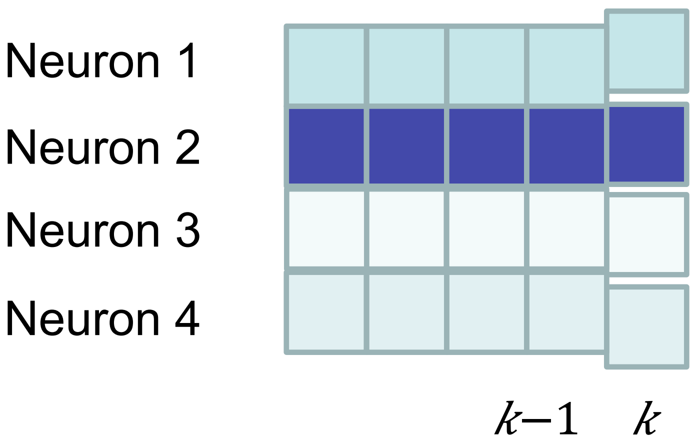
3
- **Simultaneous estimation of posterior and parameters (EM-algorithm).**

4
- **Model validation in a Bayesian framework (Bayes factor, ABIC),**

5
- Applications to **neural decoding** and **plasticity**.

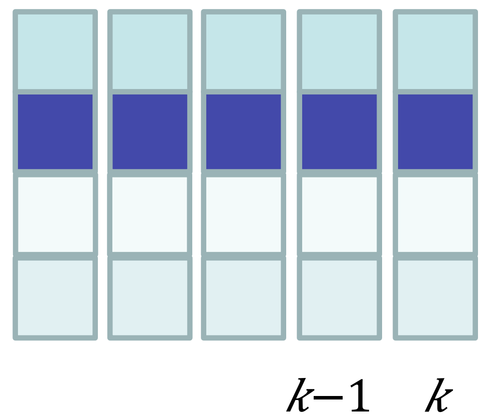
Relations between GLM and Max-ent

Bernoulli-GLM model



Conditional independence

Maximum entropy model



Joint probability mass function